The subject of event-by-event fluctuations of charged particles produced in high-energy heavy-ion collisions has recently gained considerable attention [1, 2, 3, 4, 5]. The focus has mainly been in the identification of a measure that can differentiate a quark-gluon plasma (QGP) from a hadron gas (HG). No consideration has been given to the fluctuations introduced by the phase transition (PT) itself. If the PT corresponds to the second-order chiral transition that has the usual $O(4)$ symmetry of two massless quarks, it has been argued that long-range correlation typical of critical phenomena is unlikely to occur because the only scale in the problem is characterized by the realistic pion mass that is roughly the same as the critical temperature [6]. As a result one does not expect large fluctuations, especially in the charge sector. On the other hand, if the chiral transition takes place rapidly far from thermal equilibrium, then the disorientation of the isovector order parameter can lead to long-wavelength modes and large fluctuations [7, 8, 9]. The usual signature for such disoriented chiral condensates (DCC) is the large fluctuation in the neutral-to-charge ratio of the produced particles [10, 11, 12, 13]. In this paper we study the nature of the fluctuations within the charge sector and identify measures that can be computed explicitly as well as being amenable to direct experimental verification. They do not depend on the existence of long-range correlations that may be suppressed by the rapid expansion in a heavy-ion collision.

The usual starting point of a consideration of DCC is the linear σ model, for which the potential is

$$V = \frac{\lambda}{4}(\Phi^2 - v^2)^2 - H\sigma, \tag{1}$$

where the chiral fields are represented by a vector $\Phi = (\sigma, \vec{\pi})$ in $O(4)$ space. The parameters λ, v, and H are determined by the masses m_π, m_σ and the pion decay constant f_π. At temperature T far below the critical T_c, the normal vacuum is characterized by $\langle\sigma\rangle \neq 0$, and $\langle\vec{\pi}\rangle = 0$. At T well above T_c there is approximate $O(4)$ symmetry for which $\langle\Phi\rangle = 0$. If the QCD plasma is close to thermal equilibrium as it cools from above T_c to below, the PT results in $\langle\Phi\rangle$ becoming nonzero in the σ direction with no large fluctuations expected in the angular deviation from σ. However, in the quench scenario far from thermal equilibrium [7], the plasma loses touch with the vacuum orientation and $\langle\vec{\pi}\rangle$ becomes nonzero along arbitrary directions in isospin space in different spatial regions, thereby generating large fluctuations in the charges of the pions produced. What we seek are the signatures of those fluctuations that are independent of the details of the theory, specifically, the parameters governing the chiral transition. We show the existence of a numerical index ν that can serve as a signature of DCC. As an alternative to the D measure that has been suggested in [1, 4], we consider another measure B that can clearly distinguish the different types of charge fluctuations.

It is convenient to start with the coherent-state representation for the statistical fluctuations since the multiplicity distribution of a pure coherent state $|\alpha\rangle$ is Poissonian, i.e., $|\langle n|\alpha\rangle|^2 = P_n^0$, with average multiplicity

$$\langle n\rangle = \left\langle \alpha \left| \int dz a^\dagger(z) a(z) \right| \alpha \right\rangle = \int dz \, |\alpha(z)|^2 , \tag{2}$$

where the property $a(z)|\alpha\rangle = \alpha(z)|\alpha\rangle$ has been used. We generalize this formalism by incorporating isospin and treat $\vec{\phi}$ as the eigenvalues of the isovector annihilation operators $\vec{a}(z)$ [14]. The total average density of hadrons (assumed to be pions only) is then

$$\left\langle \vec{\phi} \left| \vec{a}^\dagger(z)\vec{a}(z) \right| \vec{\phi} \right\rangle = \left|\vec{\phi}(z)\right|^2 . \tag{3}$$

The spatial coordinate z can be regarded as (pseudo) rapidity in heavy-ion collisions, as in [14], but at this point that identification is unnecessary. Our theoretical results, expressed below by Eqs. (16), (17) and (21), are independent of what z is exactly. For experimental analysis of the data, extensive discussion will be given below.

The application of coherent states and the Ginzburg-Landau formalism [15] to multiparticle production was considered many years ago [14]. We use the same approach here, as in [16, 17], to study the thermal fluctuations in the chiral transition to DCC. We take the Ginzburg-Landau free energy to be

$$F[\vec{\phi}] = \int_\delta dz \left[a \left|\vec{\phi}(z)\right|^2 + b \left|\vec{\phi}(z)\right|^4 \right], \tag{4}$$

where only the isosymmetric part of the potential in Eq. (1) is adapted here. The derivative term in (4) is neglected here, since our earlier studies of isoscalar $\phi(z)$ indicate that the inclusion of $\partial\phi(z)/\partial z$ term in the free energy leads to negligible effect on the scaling result [17, 18]. When T is lowered below T_c, a becomes negative, while b remains positive, and the system makes a transition to the hadron phase whose density fluctuates around $\left|\vec{\phi}\right|^2 = -a/2b$. The hadronic multiplicity distribution is then given by

$$P(n_+, n_-, n_0) = Z^{-1} \int \mathcal{D}\vec{\phi} \; P^0(n_+, n_-, n_0, \left|\vec{\phi}\right|^2) e^{-F[\vec{\phi}]} \tag{5}$$

where

$$Z = \int \mathcal{D}\vec{\phi}\; e^{-F[\vec{\phi}]}, \tag{6}$$

$$P^0(n_+, n_-, n_0, \left|\vec{\phi}\right|^2) = \prod_{i=+,-,0} P^0(n_i, |\phi_i|^2), \tag{7}$$

$$P^0(n_i, |\phi_i|^2) = \frac{1}{n_i!}\left(\int_\delta dz\,|\phi_i|^2\right)^{n_i} e^{-\int_\delta dz|\phi_i|^2}. \tag{8}$$

Since these quantities are meaningful only in the hadron phase, we shall in the following consider only the situation where $a < 0$.

The central aim of our proposed analysis is to find a measure of the charge fluctuations due to a chiral transition to arbitrary $\langle\vec{\pi}\rangle$ directions with the property that the measure is independent of the details, more specifically, the parameters a, b, and δ in Eq. (4). We shall assume that δ can be arranged to be small so that $\vec{\phi}$ can be regarded as constant inside δ. We then have

$$F[\vec{\phi}] = \delta\left[a\left|\vec{\phi}\right|^2 + b\left|\vec{\phi}\right|^4\right], \tag{9}$$

where the value of each of $|\phi_i|$ is allowed to vary throughout the complex plane in the functional integrals in Eqs. (5) and (6).

In our search for a quantity that is independent of δ, a, and b, we first consider the bivariate factorial moments

$$f_{q_1,q_2} = \sum_{n_+=q_1}^{\infty}\sum_{n_-=q_2}^{\infty}\sum_{n_0=0}^{\infty}\frac{n_+!}{(n_+-q_1)!}\frac{n_-!}{(n_--q_2)!}P(n_+,n_-,n_0), \tag{10}$$

for integer values of q_1 and q_2. If $P(n_+, n_-, n_0)$ were the statistical distribution given in Eqs. (7) and (8), then (10) would yield $f_{q_1,q_2} = f_{0,1}^{q_1} f_{1,0}^{q_2}$ for any q_1 and q_2. Deviation from this trivial result on account of Eqs. (5) and (8) is then a measure of the effect of the DCC on the charge fluctuations plus other effects to be discussed below. Making the appropriate substitutions we obtain

$$\begin{aligned} f_{q_1,q_2} &= Z^{-1}\delta^{q_1+q_2}\,\pi^3\int_0^\infty d\,|\phi_+|^2\int_0^\infty d\,|\phi_-|^2\int_0^\infty d\,|\phi_0|^2 \\ &\quad |\phi_+|^{2q_1}\;|\phi_-|^{2q_2}\; e^{-\delta\left(a|\vec{\phi}|^2+b|\vec{\phi}|^4\right)}. \end{aligned} \tag{11}$$

Changing the integration variables to the set $s = |\phi_+|^2, t = |\phi_+|^2 + |\phi_-|^2$, and $u = |\vec{\phi}|^2$, so that only u is integrated from 0 to ∞, we obtain a significantly simplified, closed form

$$f_{q_1,q_2} = \left(\frac{\delta}{b}\right)^{(q_1+q_2)/2}\frac{2B\,(q_1+1,q_2+1)}{q_1+q_2+2}\,\frac{J_{q_1+q_2+2}(x)}{J_2(x)}, \tag{12}$$

where $B(m, n)$ is the Euler-beta function, and

$$x = |a|\,\sqrt{\delta/b}. \tag{13}$$

The function $J_p(x)$, defined by

$$J_p(x) = \int_0^\infty du\,u^p\,e^{xu-u^2}, \tag{14}$$

can be related to the parabolic cylinder function, but is straightforwardly computable.

Although f_{q_1,q_2} has complicated dependence on δ, a and b, different (q_1, q_2) moments have the same dependence if $q_1 + q_2 \equiv q$ is the same. Furthermore, the factor $(\delta/b)^{(q_1+q_2)/2}$ is cancelled for the normalized factorial moments, which we define as

$$\begin{aligned} F_{q_1,q_2} &\equiv \frac{f_{q_1,q_2}}{f_{1,0}^{q_1} f_{0,1}^{q_2}} \\ &= \frac{2B\,(q_1+1,q_2+1)}{q_1+q_2+2}\left[\frac{3J_2(x)}{J_3(x)}\right]^{q_1+q_2}\frac{J_{q_1+q_2+2}(x)}{J_2(x)}. \end{aligned} \tag{15}$$

Evidently, $F_{q_1,q_2}(x)$ is a function of x only. Its dependence on x is shown in Fig. 1 in a log-log plot. No scaling behavior can be seen. However, if we plot $\ln F_{q_1,q_2}$ vs $\ln F_{2,2}$ as in Fig. 2, we find a substantial region in which the relationship is linear. In that linear region we can write

$$F_{q_1,q_2} \propto F_{2,2}^{\beta_{q_1,q_2}}, \tag{16}$$

which is a behavior that is independent of x. More explicitly, we can determine β_{q_1,q_2} by straightline fits of the curves in Fig. 2 in the region $0.2 < \ln F_{2,2} < 0.9$. The result yields β_{q_1,q_2} as a function of only the sum, $q_1 + q_2 = q$, as can be seen directly from Eq. (15). That dependence of β_q on q is shown in Fig. 3. Apart from the point for $q = 2$, it is a linear dependence of $\ln \beta_q$ on $\ln(q-1)$. Thus we can write

$$\beta_q \propto (q-1)^{\nu}, \qquad \nu = 1.29. \tag{17}$$

This index ν is independent of x, and therefore of δ, a and b (so long as $a < 0$). The scaling behaviors (16) and (17) summarize the properties of charge fluctuations in a chiral transition to DCC and culminate in a numerical index ν that characterizes the phenomenon.

To verify the above behavior experimentally, it is necessary to vary x, which is the implicit variable in Eq. (16). Since a and b in Eq. (13) are not subject to experimental control, only δ can be varied. A more extensive discussion of the experimental cuts that are optimal for detecting the signal is postponed until another prediction independent of x is presented.

A few remarks should first be made regarding the sensitivity of the above theoretical result to statistical fluctuations in the experimental background. It was shown by Białas and Peschanski [19] that the factorial moments filter out the statistical fluctuations represented by the Poissonian distribution in Eq.(8). However, if the DCC is produced in a background of conventionally produced hadrons, we have to consider an additional contribution to the mean multiplicity. Thus we replace $|\phi_i|^2$ on the RHS of (8) by $|\phi_i|^2 + S$, where S denotes the mean density of the statistical background. If there are statistical fluctuations in the orientations of the fields after the chiral transition, we shall let S represent them also. We ask whether our result is sensitive to a small perturbation by S. Clearly, if S were large, it would be hard to find the signature of DCC in the presence of a large non-DCC hadronization process. If S is small, we can carry the above theoretical calculation to first order in S, and find that a term must be added to F_{q_1,q_2} in (15), which is proportional to $S\sqrt{b\delta}$. At small δ the effect of S is expected to be negligible.

There is, however, a limitation on how small δ should be allowed to be. That can be seen from Eq.(11) where the exponential damping term becomes ineffective at small δ. Without letting $|\vec{\phi}|^2$ become large enough to require higher-order terms in the Ginzburg-Landau free energy, a rough lower bound on δ is derived in Ref. [16] and is equally applicable here; it is $\delta > x_0^2 b/a^2$, where $x_0 = \sqrt{4\ln 2} = 1.67$. That bound translates to $S\sqrt{b\delta} > (Sb/|a|)x_0$. Thus the sensitivity to the statistical background depends on its strength relative to that of the dynamical strength $|a|/b$, as is reasonable. Our conclusion is therefore that unless the mean multiplicity of the statistical background is small compared to that of the DCC production, our approach of searching for scaling behavior would not be successful.

To put the curves in Fig.2 to experimental test, one inevitably has to deal with the inaccuracies in the determination of F_{q_1,q_2}, and in particular with $F_{2,2}$. If the error bars are large, the effectiveness of this approach is clearly limited. Even if they are not large, the data points may admit a larger scaling region than what we have consider in Fig.2 in the derivation of $\nu = 1.29$. In fact, our lower bound on δ mentioned above implies that the curves in Fig.1 are reliable only for $x < x_0$, or $-\ln x < -0.51$. It translates to a restriction on the range of $F_{2,2}$ to the region $\ln F_{2,2} < 0.65$ in Fig. 2. Thus fitting the curves in that region by straight lines with allowance for errors we find that it is necessary to modify the value of ν to

$$\nu = 1.42 \pm 0.13. \tag{18}$$

It is clear that we can no longer claim strict x-independence in our result, when the various complications discussed above are taken into account. However, despite the limitations arising from theoretical considerations, we feel that the proposal for the experimental measurement of F_{q_1,q_2} may nevertheless yield interesting insight into the formation of DCC.

To reduce the sensitivity to the details in the scaling analysis, we now consider a global measure that makes contact with the observables proposed in [1, 2, 3, 4, 5]. Let us first list the simple identities:

$$\begin{gathered} f_{1,0} = \langle n_+ \rangle , \quad f_{0,1} = \langle n_- \rangle , \quad f_{1,1} = \langle n_+ n_- \rangle , \\ f_{2,0} = \langle n_+(n_+ - 1) \rangle , \quad f_{0,2} = \langle n_-(n_- - 1) \rangle . \end{gathered} \tag{19}$$

In terms of the usual definitions, $N_{\rm ch} = n_+ + n_-$, $Q = n_+ - n_-$, and $\langle \delta X^2 \rangle = \langle X^2 \rangle - \langle X \rangle^2$, we then have

$$\langle \delta Q^2 \rangle = 2\left(f_{2,0} + f_{1,0} - f_{1,1}\right), \tag{20}$$

where the symmetry $f_{i,j} = f_{j,i}$ has been used. Let us now consider the quantity

$$B = \frac{\langle \delta Q^2 \rangle - \langle N_{\rm ch} \rangle}{\langle n_+ n_- \rangle}. \tag{21}$$

Using Eqs. (19) and (20), together with (12) we obtain

$$B = 2\left(\frac{f_{2,0}}{f_{1,1}} - 1\right) = 2\left[\frac{B(3,1)}{B(2,2)} - 1\right] = 2. \tag{22}$$

On the other hand, in terms of D [1, 4], where

$$D = 4\left\langle \delta Q^2 \right\rangle / \left\langle N_{\rm ch} \right\rangle, \tag{23}$$

Eq. (21) implies

$$B = \frac{D/4 - 1}{\langle n_+ n_- \rangle / \langle N_{\rm ch} \rangle}. \tag{24}$$

It has been argued that $D \leq 4$ whether the thermal system is a QGP or a HG [1], so in that scenario we have $B \leq 0$ whatever $\langle n_+ n_- \rangle / \langle N_{\rm ch} \rangle$ may be. That prediction is distinctively different from 2 in our scenario. Qualitatively speaking, the charge fluctuations in chiral transition to DCC are much greater than what can be expected from the thermal fluctuations in either the QGP or the HG phases. Thus the measurement of B can decisively determine the nature of the PT that the QGP system undergoes. The presence of statistical fluctuations in the background will undoubtedly weaken this claim to an extent that is proportional to the strength of such fluctuations, as we have found in the study of the scaling behavior.

In order that our theoretical prediction can be applied to the data, it is important that the data are analyzed in the proper way, which is the subject we now address. The factorial moments can reveal scaling behavior if they can capture the rare events with large spikes without being overwhelmed by the contributions from average multiplicities [19]. In other words, if we, for simplicity, consider a single factorial moment

$$f_q = \sum_{n=q}^{\infty} n(n-1)\cdots(n-q+1)P_n, \tag{25}$$

we see that f_q probes the high end of the distribution P_n with $n \geq q$; thus if $q \gg \langle n \rangle$, only the less frequent events with $n \gg \langle n \rangle$ can contribute to Eq. (25). That can be achieved for heavy-ion collisions either by using extremely high q, or by making severe cuts in data selection to reduce $\langle n \rangle$. Neither was done in previous analyses of the nuclear data and nothing of interest was found in F_2 or F_3 in the bulk data.

To test the power-law behaviors (16)–(19) and the prediction $B = 2$, it is necessary to make cuts in the data to best reveal the features of this analysis. To see what cuts to make, we first discuss multiparticle production in heavy-ion collisions. If the quark-hadron PT is first-order, then what we have considered here is irrelevant. For a second-order chiral transition it is likely that, if the DCCs are to be created at all, they would appear as clusters of hadrons in different regions in space and time amidst other patches of hadrons produced as a result of the gradual cooling of the QGP that undergoes the normal critical transition. If it is possible to restrict the observation to a very short duration in the emission time, then we should see many regions of voids where no particles are produced, separating the regions of particles produced either as oriented or disoriented condensates. Unfortunately, a selection in emission time is not experimentally feasible. The event multiplicity integrates over all times and smooths out the fluctuations in clusters and voids. Thus to detect the DCC by our fluctuation analysis it is necessary to make a severe cut in p_T and select only the particles emitted into a narrow p_T window. Such a Δp_T cut achieves two goals: one, to reduce the multiplicities to facilitate the factorial moment analysis discussed above; two, to minimize the overlap of hadron emissions at different times. The correlation between p_T and the emission time is discussed in Ref.[20], where a simulation is done to exhibit the void patterns.

Now, suppose that in a narrow p_T window n_+ and n_- charged particles are collected in $\Delta\phi$ in azimuthal angle and $\Delta y = \delta$ in rapidity. What we suggest is that $\langle N_{\rm ch} \rangle$ be only about 2 so that F_{q_1,q_2} can be calculated for $N_{\rm ch} \geq q = q_1 + q_2$ up to q about 10, as shown in Fig. 1. Such values of $N_{\rm ch}$ are large deviations from $\langle N_{\rm ch} \rangle$, but represent only a tiny fraction of the total multiplicity produced in a typical event in heavy-ion collisions. Thus the problem of overlap from different emission times is minimized, while the characteristics of charge fluctuations that convey the nature of the chiral transition are retained. Of course, $\langle N_{\rm ch} \rangle$ depends on the sizes of the cuts. But our theoretical analysis based on the Ginzburg-Landau formalism indicates that the scaling exponent β_{q_1,q_2}, the index ν, and the quotient B are

all independent of the window sizes, so long as there is enough statistics to render accurate determination of f_{q_1,q_2}. Since the charge fluctuations of DCC are much larger than those among the hadrons produced in the normal critical transition, the contribution of the latter to our measure is expected to be negligible.

Recently, WA98 presented their data that reveal no correlated charge-neutral fluctuations in Pb-Pb collisions [21]. Unfortunately, they have no information on p_T and cannot make the p_T cut that we regard as essential. The absence of any signature of DCC in the bulk events does not preclude the possibility of local condensates of the type we consider here.

It is clear from Eqs. (20) and (21) that B involves order-sum q not greater than 2, whereas the power-law behavior of β_q shown in Fig. 3 is for $q > 2$. Although the former seems simpler to investigate, the smallness of q requires $\langle N_{\rm ch}\rangle \ll 1$ in order for the factorial moments to exhibit the fluctuation properties at the far edge of the multiplicity distributions. That in turn requires the kinematical cuts to be very severe so that only in rare events can $N_{\rm ch}$ exceed q. As a consequence, the statistical errors may be large. For the latter study of β_q, the values of q are larger, so the cuts can be less severe to allow $\langle N_{\rm ch}\rangle$ to be larger. The experimental errors on β_q then may hopefully be small enough to render the determination of ν feasible. Whether q is small or large, our analysis probes large fluctuations from the mean.

Despite the many complications that accompany heavy-ion collisions, our analysis suggests a possible way of finding the DCC. If only the bulk events are examined, it is quite possible that the DCC, even if created, might escape detection. The simple consideration done here based on the essentials of chiral transition offers a potentially effective tool to initiate a first-round experimental exploration. Since the discovery of a DCC would provide a definitive confirmation of the accepted ideas of chiral dynamics in a dense system, such an exploration is well worth undertaking.

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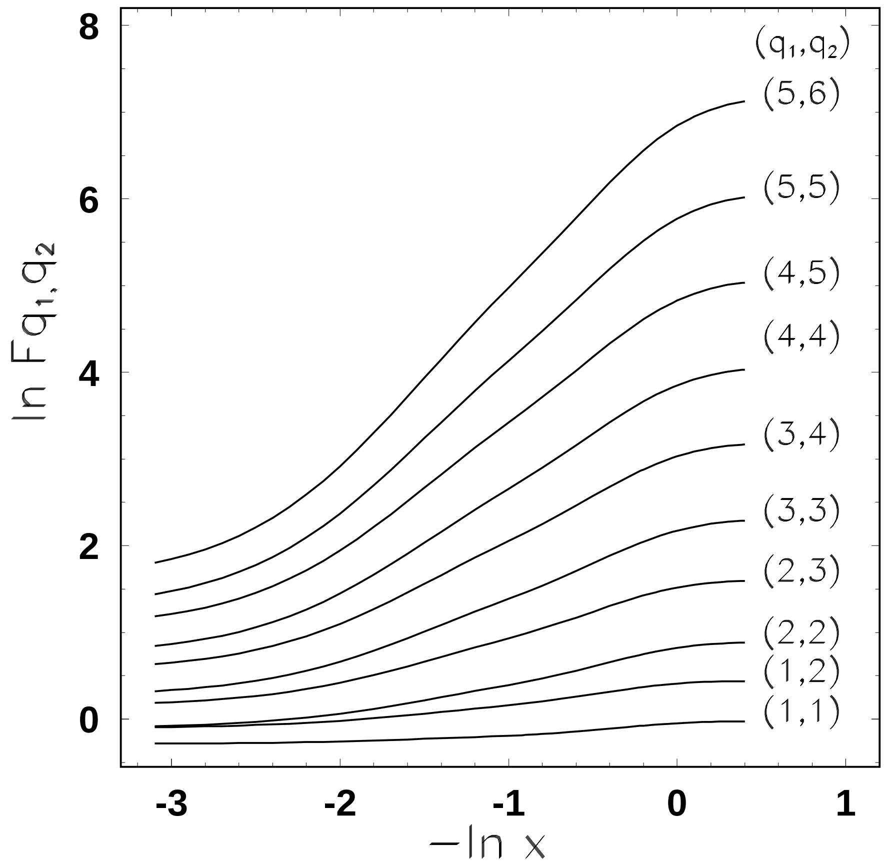

FIG. 1: F_{q_1,q_2} vs x for various orders of the moments.

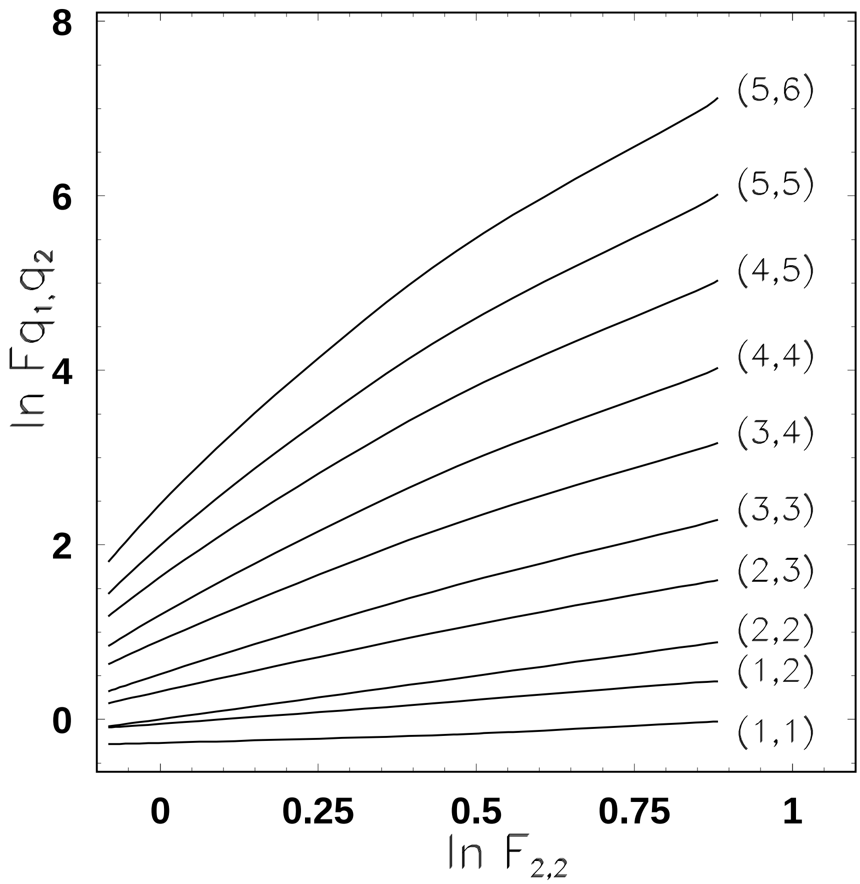

FIG. 2: F_{q_1,q_2} vs $F_{2,2}$ in log-log plot.

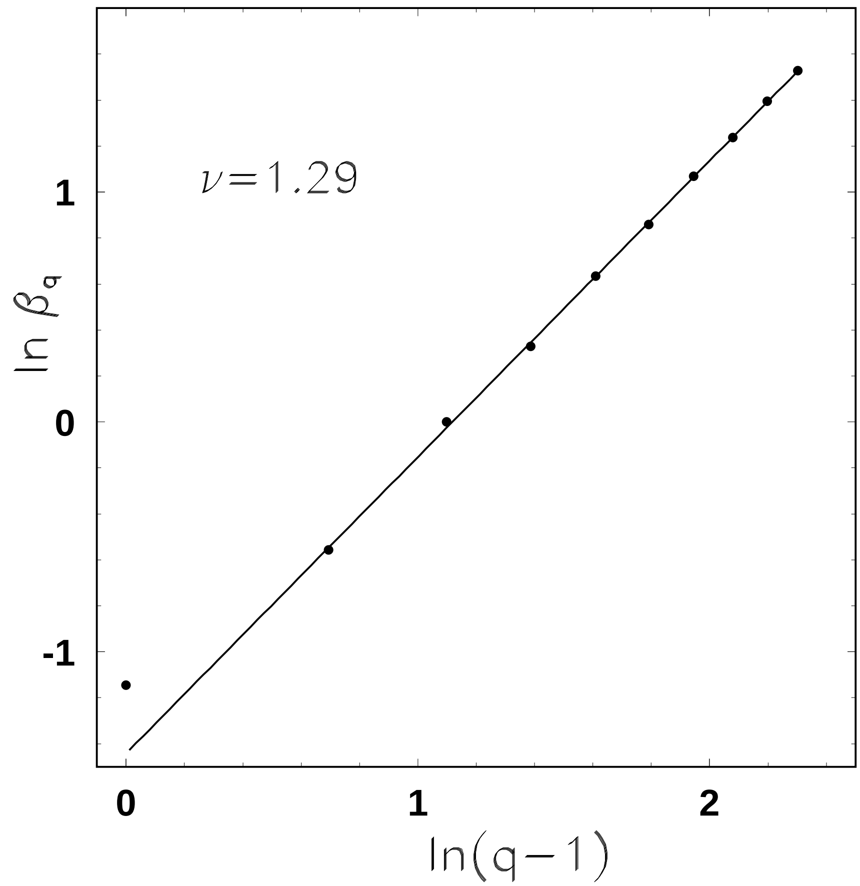

FIG. 3: Power-law behavior of β_q.